DO NOW

Find Worksheet 4.4

We need to discuss #19 and 20

4.5 Limits at Infinity

Definition of Limits at Infinity:

Let f be a function defined on some interval $(a, +\infty)$. Then:

$$\lim_{x\to\infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

Definition of Horizontal Asymptote:

The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either:

$$\lim_{x\to\infty} f(x) = L \quad \text{or} \quad \lim_{x\to-\infty} f(x) = L$$

Limits at Infinity:

If r is a positive rational number and c is any real number, then:

$\lim_{x\to\infty} \frac{c}{x^r} = 0$ and $\lim_{x\to-\infty} \frac{c}{x^r} = 0$

$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^{-x} = 0$

Examples: Find the limit.

1. $\lim_{x\to\infty} -4 + \frac{3}{x^2}$

$-4 + 0$

$\boxed{-4}$

2. $\lim_{x\to\infty} \frac{5}{e^x}$

$\lim_{x\to\infty} 5e^{-x}$

$5(0)$

$\boxed{0}$

Find the limit: 3. $\lim_{x\to\infty} \frac{3x+2}{x-4}$

$\frac{3(\infty)+2}{\infty-4}$

$\frac{\infty}{\infty}$

Indeterminate Form: $\frac{\infty}{\infty}$

One way to resolve this problem is: to divide each term in the numerator and denominator by the highest power of x found in the denominator.

Find the limit: $\lim_{x\to\infty} \frac{3x+2}{x-4}$

$\lim_{x\to\infty} \frac{\frac{3x}{x}+\frac{2}{x}}{\frac{x}{x}-\frac{4}{x}}$

$\lim_{x\to\infty} \frac{3+\frac{2}{x}}{1-\frac{4}{x}}$

$\frac{3+0}{1-0}$

$\boxed{3}$

Examples:

4. $\lim_{x\to\infty} \frac{2x+5}{3x^2+1} = \lim_{x\to\infty} \frac{\frac{2x}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{\frac{2}{x}+\frac{5}{x^2}}{3+\frac{1}{x^2}} = \frac{0+0}{3+0} = \boxed{0}$

5. $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1} = \lim_{x\to\infty} \frac{2+\frac{5}{x^2}}{3+\frac{1}{x^2}}$

$\boxed{\frac{2}{3}}$

6. $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1} = \lim_{x\to\infty} \frac{2x+\frac{5}{x^2}}{3+\frac{1}{x^2}}$

$\frac{\infty}{3} = \boxed{\infty}$

Guidelines for Finding Limits at ±∞ of Rational Functions

1. degree of numerator < degree of denominator

   the limit = 0.

2. degree of numerator = degree of denominator

   the limit is the ratio of the leading coefficients

3. degree of numerator > degree of denominator

   the limit does not exist (∞, or $-\infty$)

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Find the horizontal asymptote, if any, of each rational function.

7. $\frac{9x^2}{3x^2+1}$

   $\lim_{x\to\infty} \frac{9x^2}{3x^2+1}$

   3

   $\boxed{y=3}$

8. $\frac{9x}{3x^2+1}$

   $\lim_{x\to\infty} \frac{9x}{3x^2+1}$

   0

   $\boxed{y=0}$

9. $\frac{9x^3}{3x^2+1}$

   $\lim_{x\to\infty} \frac{9x^3}{3x^2+1}$

   ∞

   $\boxed{\text{Does not exist}}$

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HOMEWORK

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